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ON THE DETERMINATION OF SEDIMENTATION RATE OF A HOMOGENEOUS SUSPENSION

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The problem of sedimentation of a homogeneous suspension of spherical drops at low Reynolds numbers and moderate concentrations of the dispersed phase is considered. Equations that define the flow past a single drop of a system are derived using the theory of generalized functions and of averaging over the ensemble of positions of particle centers, with the binary correlation function derived in [1] taken into account. These equations make possible the determination of the force acting on an individual particle and the rate of their sedimentation depending on their volume concentration.

A considerable number of publications deals with the effect of volume concentration on the precipitation rate of a suspension of spherical particles (see the survey in [2]). The majority of theoretical investigations is based on the assumption of Stokes flow, the results of which relate to low volume concentration of the dispersed phase. Comparison of calculation results with known experimental data considered in [3] is only possible for moderately concentrated suspensions.

Theoretical determination of the precipitation rate of spherical drop suspension of moderate and high volume concentrations necessitates the knowledge of the binary correlation function structure.

An attempt was made in [4] at the determination of the precipitation rate of a diluted suspension with allowance for hydrodynamic interaction between solid spheres. The use in that work of the solution of the problem about the interaction between two isolated spheres seems to be insufficiently substantiated, since the interaction between two spheres in a diluted suspension is somewhat different from that between two isolated spheres. Generalization of results obtained for moderately concentrated suspensions by analyzing the interaction between three or more particles is, apparently, impossible owing to considerable mathematical difficulties.

The binary correlation function used in calculations in the majority of investigations was in the form of a step, which is a zero approximation and is only suitable for defining the properties of diluted systems. An exception is the investigation in which the binary correlation function was determined by the solution of the Liouville equation for a pair of isolated spheres in a pure deformation stream [5]. A very rough approximation of the binary function, suitable only for qualitative analysis was used in [6].

1. The flow around a spherical in a translational stream. Let us consider the problem of flow of a uniform translational stream of viscous incompressible fluid, defined by conventional Stokes linear equations, past a spherical particle of radius a. We locate the origin of a Cartesian system of coordinates at the particle center, with the direction of the z-axis coinciding with the direction of the fluid velocity vector away from the particle. We, also, introduce a system of spherical coordinates (r, θ, φ) attached to the particle center; θ is the angle between the radius vector x and the positive direction of the z-axis. The equations for the fields of velocity $\mathbf{v}$ and pressure p are of the form

$$\mu\Delta\mathbf{v} = \nabla p, \quad \operatorname{div}\mathbf{v} = 0, \ |\mathbf{x}| > a \qquad (1.1)$$
$$\mathbf{v} \to \mathbf{U}_0, \quad |\mathbf{x}| \to \infty$$

Below we use dimensionless variables defined by formulas

$$\mathbf{u} = \frac{\mathbf{v}}{U_0}, \quad \mathbf{r} = \frac{\mathbf{x}}{a}, \quad P = \frac{ap}{\mu U_0}, \quad \mathbf{u}_0 = \frac{\mathbf{U}_0}{U_0} \qquad (1.2)$$

Solution of Eqs. (1.1) converted to dimensionless form that, in particular corresponds to a uniform translational stream at infinity, can be expressed as follows:

$$\mathbf{u} = \mathbf{u}_0 F'(r) + (\mathbf{u}_0\cdot\mathbf{r})\,\mathbf{r}G'(r), \quad P = (\mathbf{u}_0\cdot\mathbf{r})\,H'(r) \qquad (1.3)$$
$$F'(r) = 4A'r^2 + 2B' + \frac{C'}{r} - \frac{D'}{r^3}$$
$$G'(r) = -2A' + \frac{C'}{r^3} + \frac{3D'}{r^5}, \quad H'(r) = 20A' + \frac{2C'}{r^3}$$

where A', B', C', and D' are some constants determined by related boundary conditions. The condition at infinity can be satisfied by selecting $A' = 0$ and $B' = {}^1/_2$. However for the subsequent definition of the averaged fluid motion in the particle neighborhood in the suspension the more general formulas adduced above will be required for $\mathbf{u}$ and P.

Fields $\mathbf{u}$ and P specified by formulas (1.3) identically satisfy Eqs. (1.1) in dimensionless form throughout the region $r > 0$. However in the extended region that includes point $r = 0$ the result of applying the operations Δ, div and ∇ to (1.3) is as follows:

$$\Delta u_i - \frac{\partial P}{\partial x_i} = -8\pi C' u_{0i}\delta(\mathbf{r}) - 4\pi u_{0j} D' \frac{\partial^2\delta(\mathbf{r})}{\partial x_i \partial x_j} \qquad (1.4)$$

$$\operatorname{div}\mathbf{u} = -4\pi D' u_{0j}\frac{\partial\delta(\mathbf{r})}{\partial x_j} \qquad (1.5)$$

where $\delta(\mathbf{r})$ is a three-dimensional delta function and x_i are Cartesian coordinates.

The appearance of terms D' in the right-hand sides of Eqs. (1.4) and (1.5) is due to the finiteness of the particle radius. If point particles are considered, as in [7], these terms vanish and instead of (1.5) we have $\operatorname{div}\mathbf{u} = 0$. Note that the difference between formulas (1.4) and (1.5) and the similar formulas in [7] is due to the artificial choice in [7] of velocity and pressure fields so as to identically satisfy the equation of continuity in the set of generalized functions.

Taking into account (1.5) and the relationships $\operatorname{rot}\operatorname{rot}\mathbf{u} = \nabla\operatorname{div}\mathbf{u} - \Delta\mathbf{u}$, Eq. (1.4) can be reduced to the form

$$\operatorname{rot}\operatorname{rot}\mathbf{u} + \nabla P = 8\pi C^1\mathbf{u}_0\delta(\mathbf{r}) \qquad (1.6)$$

2. Correlation functions. The device of correlation functions is used below for the derivation of averaged equations that define the interaction between the sample point and the surrounding fluid. We denote by $\mathbf{R}_N$ the set of radius vectors $(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)$ of particle centers, by $f_N(\mathbf{R}_N; 0)$ the N-partial conditional correlation function, and by $f_1(\mathbf{r})$, $f_2(\mathbf{r}_1, \mathbf{r}_2)$ and $f_1(\mathbf{r}_1; \mathbf{r}_2)$ the unary, binary, and the conditional unary correlation functions, respectively. According to the theorem of multiplication of probabilities

$$f_2(\mathbf{r}_1, \mathbf{r}_2) = f_1(\mathbf{r}_2) f_1(\mathbf{r}_1; \mathbf{r}_2) \tag{2.1}$$

$$f_N(\mathbf{R}_N; 0) = f_1(\mathbf{r}_1; 0) f_{N-1}(\mathbf{R}_{N-1}; \mathbf{r}_1, 0)$$

Various methods of correlation function normalization are available. Here we assume that the equalities

$$\frac{1}{V^N}\int f_N(\mathbf{R}_N; 0)\, d\mathbf{R}_N = 1 \tag{2.2}$$

$$\frac{1}{V^{N-1}}\int f_{N-1}(\mathbf{R}_{N-1}; \mathbf{r}_1, 0)\, d\mathbf{R}_{N-1} = 1$$

are satisfied, i.e. the introduced functions are dimensionless.

Since the system considered here is spatially homogeneous $f_1(\mathbf{r}) \equiv 1$ and, in conformity with (2.1), the binary correlation function is the same as the conditional unary function. Furthermore we assume that the particle distribution in the neighborhood of the sample particle is spherically symmetric. Hence

$$f_2(\mathbf{r}_1, \mathbf{r}_2) = g(r_{12}), \quad r_{12} = |\mathbf{r}_1 - \mathbf{r}_2|$$

Function $g(r)$ depends only on distance between particle centers and is independent of angle variables.

3. Derivation of averaged equations. Let us consider the sedimentation in a suspension of $N + 1$ $(N \gg 1)$ spherical drops of fluid of viscosity μ'' in a fluid of viscosity μ. The sedimentation rate depends on the dimension and shape of particles, the difference between the weight and the ejecting force, and also on the volume concentration of particles. In the case considered here of spherical particles the sedimentation rate is proportional to excess of weight, and otherwise depends on the volume concentration [4]. We assume that the suspension is statistically homogeneous and contained in volume V whose characteristic linear dimension considerably exceeds the average distance between particles. We choose a sample particle and locate the origin of a Cartesian coordinate system at the center of that particle. We denote by U the velocity of the stream flowing onto the sample particle in a system of point particles.

The flow around the sample particle is analyzed using the dimensionless variables defined by formulas (1.2) in which the so far unknown velocity U is substituted for velocity U_0. The velocity and pressure fields in the sample particle neighborhood are assumed to be of the form (1.3) as in the case of a single particle. But the coefficients A', B', C', and D' are considered to be known beforehand and dependent on the volume concentration of particles.

Using Eqs. (1.5) and (1.6) we can write

$$\operatorname{rot}\operatorname{rot}\mathbf{u}+\nabla P=8\pi\sum_{n=1}^{N}C_n'u_0\delta(\mathbf{r}-\mathbf{r}_n)$$
$$\operatorname{div}\mathbf{u}=-4\pi\sum_{n=1}^{N}D_n'u_{0j}\frac{\partial}{\partial x_j}\delta(\mathbf{r}-\mathbf{r}_n) \tag{3.1}$$

where $\mathbf{r}$ is the observed point and $\mathbf{r}_n$ are points of particle centers. In a statistically homogeneous system the coefficients C_n' and D_n' are the same for all particles and equal to C' and D', respectively.

We average Eqs. (3.1) using function $f_N(\mathbf{R}_N;\,0)$ and taking into account properties of correlation functions (2.1) and (2.2).

In the right-hand sides of (3.1) we have

$$\left\langle\sum_{n=1}^{N}\delta(\mathbf{r}-\mathbf{r}_n)\right\rangle=\frac{1}{V^N}\sum_{n=1}^{N}\int f_N(\mathbf{R}_N;\,0)\,\delta(\mathbf{r}-\mathbf{r}_n)\,d\mathbf{R}_N= \tag{3.2}$$
$$\frac{1}{V^N}\sum_{n=1}^{N}\int f_1(\mathbf{r}';\,0)\,f_{N-1}(\mathbf{R}_{N-1};\,\mathbf{r}',0)\,\delta(\mathbf{r}-\mathbf{r}_n)\,d\mathbf{R}_N=$$
$$n\int f_1(\mathbf{r}';\,0)\,\delta(\mathbf{r}-\mathbf{r}')\,d\mathbf{r}'=ng(r)$$
$$\left\langle\sum_{n=1}^{N}\mathbf{u}_0\cdot\nabla\delta(\mathbf{r}-\mathbf{r}_n)\right\rangle=n\mathbf{u}_0\nabla g(r)$$

where (and in what follows) angle brackets denote quantities averaged over the ensemble.

Note that formulas (3.2) are in no way based on any assumptions as regards low volume concentration of particles.

The substitution of points for particles must be taken into account by suitable equations for the determination of the binary correlation function.

The binary correlation function $g(r)$ is discontinuous at $r=2$. Using the rule of differentiation of discontinuous functions [8] we obtain

$$(\mathbf{u}_0\cdot\nabla g)=(\mathbf{u}_0\cdot\mathbf{n})\frac{dg}{dr}+[g]_S(\mathbf{u}_0\cdot\mathbf{n})\,\delta(r-2) \tag{3.3}$$

where $\mathbf{n}$ is the unit vector of the outer normal of the sphere concentric with the sample particle, $[g]_S=g(2)$ is the jump of the g-function at passing the sphere $r=2$, and $\delta(r-2)$ is a univariate delta function.

Using (3.2) and (3.3) we obtain from Eqs. (3.1) in region $r\geqslant 1$ the following equations;

$$\operatorname{rot}\operatorname{rot}\langle\mathbf{u}\rangle+\nabla\langle P\rangle=6cC'\mathbf{u}_0 g(r) \tag{3.4}$$
$$\operatorname{div}\langle\mathbf{u}\rangle=-3cD'(\mathbf{u}_0\cdot\mathbf{n})\left[\frac{dg}{dr}+g(2)\,\delta(r-2)\right] \tag{3.5}$$

The surface of sphere $r=2$ is the discontinuity surface of the particle number density. In region $1<r<2$ that does not contain centers of particles surrounding the sample, Eqs. (3.4) and (3.5) reduce to ordinary Stokes equations. In region $r>2$ the right-hand sides of (3.4) and (3.5) retain the terms which define the effect of particles whose centers are in that region. Equations (3.4) and (3.5) define the motion of an individual drop in a certain effective flow field produced by surrounding particles.

Below we denote quantities in region $1 < r < 2$ by a prime, while those in region $r > 2$ appear without primes. It follows from Eq. (3.5) that the radial component of the averaged velocity vector

$$\langle u_r \rangle - \langle u_r' \rangle = -3cD'g\,(2) \tag{3.6}$$

is discontinuous at the surface of sphere $r = 2$.

Note that the appearance of discontinuities in the velocity and stress fields at the discontinuity surface of quantity c in suspensions of solid particles was indicated in [9], where a different problem of the motion of neutral floating particles free of the action of external forces was considered.

Equation (3.4) implies that the averaged pressure and vorticity $\operatorname{rot}\langle \mathbf{u} \rangle$ are everywhere, including the surface $r = 2$, continuous, hence

$$\langle P \rangle = \langle P' \rangle, \quad \operatorname{rot}\langle \mathbf{u} \rangle = \operatorname{rot}\langle \mathbf{u}' \rangle, \quad r = 2 \tag{3.7}$$

To prove these relationships we write Eq. (3.4) in the form of projections on the basis vectors of the spherical system of coordinates

$$\frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\operatorname{rot}\langle \mathbf{u} \rangle)_\varphi \sin\theta + \frac{\partial\langle P \rangle}{\partial r} = 6cC'g\cos\theta \tag{3.8}$$

$$\frac{1}{r}\frac{\partial}{\partial r}r(\operatorname{rot}\langle \mathbf{u} \rangle)_\varphi - \frac{1}{r}\frac{\partial\langle P \rangle}{\partial\theta} = 6cC'g\sin\theta$$

The second of Eqs. (3.8) makes it possible to state that $\operatorname{rot}\langle \mathbf{u} \rangle$ is continuous, if the pressure is continuous or, when discontinuous, remains bounded. But then in conformity with the first of these equations it is possible to establish the continuity of pressure $\langle P \rangle$.

The continuity of vorticity at the surface $r = 2$ implies the continuity of the tangential velocity component at that surface

$$\langle u_\theta \rangle = \langle u_\theta' \rangle, \quad r = 2 \tag{3.9}$$

Away from the sample particle we have

$$\langle \mathbf{u} \rangle \to \mathbf{u}_0, \quad r \to \infty \tag{3.10}$$

which corresponds to a homogeneous stream flowing onto the particle.

4. Determination of the sedimentation rate of suspension. Let us consider the problem of the flow of a fluid defined by Eqs. (3.4) and (3.5) around a spherical drop. The quantities related to the fluid inside the drop will be denoted by two primes. These quantities are reduced to the dimensionless form by formulas (1.2), except that pressure $\langle P'' \rangle$ is related to μ'' instead of μ. The equations of motion of fluid inside the drop are similar to Eqs. (1.1), i.e.

$$\Delta\langle \mathbf{u}'' \rangle = \nabla\langle P'' \rangle, \quad \operatorname{div}\langle \mathbf{u}'' \rangle = 0 \tag{4.1}$$

and the boundary conditions at the drop surface $r = 1$ are

$$\langle u_r'\rangle = \langle u_r''\rangle = 0, \quad \langle u_\theta'\rangle = \langle u_\theta''\rangle \tag{4.2}$$
$$\sigma\left(\frac{\partial}{\partial r}\langle u_\theta''\rangle - \langle u_\theta''\rangle\right) = \frac{\partial}{\partial r}\langle u_\theta'\rangle - \langle u_\theta'\rangle, \quad \sigma = \frac{\mu''}{\mu}$$

The solution of Eqs. (3.4) and (3.5) that correspond to a homogeneous translational flow at infinity is of the form (1.3). Similarly for $r < 1$

$$\langle \mathbf{u}''\rangle = \mathbf{u}_0 F''(r) + (\mathbf{u}_0\cdot\mathbf{r})\,\mathbf{r}G''(r) \tag{4.3}$$
$$\langle P''\rangle = (\mathbf{u}_0\cdot\mathbf{r})\,H''(r)$$
$$F''(r) = 4A''r^2 + 2B'', \quad G''(r) = -2A'', \quad H''(r) = 20A''$$

where it is taken into account that $|\mathbf{u}''| < \infty$ when $r = 0$.

In region of $r > 2$ the fields $\langle \mathbf{u}\rangle$ and $\langle P\rangle$ of the form (1.3) and (4.3) reduce Eqs. (3.4) and (3.5) to the following system of ordinary differential equations for the determination of functions F, G, and H:

$$\frac{d^2F}{dr^2} + \frac{2}{r}\frac{dF}{dr} + 2G - H = -6cC'g - 3cD'\frac{1}{r}\frac{dg}{dr} \tag{4.4}$$
$$\frac{dG}{dr} - \frac{dH}{dr} = \frac{1}{r}\frac{d^2F}{dr^2} - \frac{1}{r^2}\frac{dF}{dr}$$
$$\frac{1}{r}\frac{dF}{dr} + 4G + r\frac{dG}{dr} = -3cD'\frac{1}{r}\frac{dg}{dr}$$

The general solution of the linear nonhomogeneous system of Eqs. (4.4) with allowance for $G(r) \to 0$ when $r \to \infty$ is of the form

$$F(r) = 2B + \frac{C}{r} - \frac{D}{r^3} - 3cD'(g-1) - \int_2^r \frac{R_1(r)}{2r^2}\,dr - \int_2^r \frac{R_2(r)}{2r^4}\,dr \tag{4.5}$$
$$G(r) = \frac{C}{r^3} + \frac{3D}{r^5} + \frac{R_1(r)}{2r^3} - \frac{R_2(r)}{2r^5}$$
$$H(r) = 6cC' + \frac{2C}{r^3} + 3cD'\frac{1}{r}\frac{dg}{dr} + \frac{R_1(r)}{r^3}$$
$$R_1(r) = \int_2^r r^2 Q(r)\,dr, \quad R_2(r) = \int_2^r r^4 Q(r)\,dr$$
$$Q(r) = 6cC'[g(r) - 1] - 3cD'\left(\frac{dg^2}{dr^2} + \frac{2}{r}\frac{dg}{dr}\right)$$

Taking into account that $g(r)$ tends exponentially to unity when $r \to \infty$ from condition (3.10) we obtain the constant B

$$B = \frac{1}{2} + 2cC'I + cD'\left[g(2) - 1 + 2\frac{dg(2)}{dr}\right] \tag{4.6}$$
$$I = \int_2^\infty r[g(r) - 1]\,dr$$

Boundary conditions (4.2) and conditions (3.6), (3.7), and (3.9) at the discontinuity surface of the particle number yield for the determination of unknown coefficients in the velocity and pressure fields the following system of linear equations:

$$A' + B' + C' + D' = 0, \quad 4A' + 2B' + C' - D' = v_0 \tag{4.7}$$
$$16A' + 2B' + {}^1/_2 (C' - C) - {}^1/_8 (D' - D) = 2B - 3cD' [g(2) - 1]$$
$$2A' - {}^1/_8 (C' - C) - {}^3/_{32} (D' - D) = -{}^3/_4 cD'g(2)$$
$$20A' + {}^1/_4 (C' - C) = 6cC' + {}^3/_2 cD'g(2)$$
$$20A' - \frac{1}{2}(C' - C) = -3cD' \frac{dg(2)}{dr}$$
$$A'' + B'' = 0, \quad 4A'' + 2B'' = v_0$$
$$\sigma A'' = A' + D', \quad \langle u_\theta'' \rangle = -v_0 \sin\theta$$

where the supplementary quantity v_0 relates to the tangential velocity component on surface $r = 1$.

Solution of the system of Eqs. (4.7) yields

$$C' = -\frac{3 - 2v_0 - v_0 c}{4 - 23c + 12cI + 0.4c^2} \tag{4.8}$$
$$D' = -{}^1/_3 (1 - {}^2/_5 c) C' - {}^1/_3 v_0$$
$$v_0 = \frac{1}{2}(1 - c)\left[1 - 4c + 2cI - \frac{1}{10}c^2 + \sigma\left(1 - \frac{23}{4}c + 3cI + \frac{1}{10}c^2\right)\right]^{-1}$$

For the determination of force F acting on the particle it is necessary to know only C', since $F = -8\pi C'$ to which in dimensional quantities corresponds $F = -8\pi\mu a C' U$. That force is equal to the drag of a single settling particle

$$F = 2\pi\mu a U_0 (2 + 3\sigma) / (1 + \sigma)$$

Comparison of these formulas shows that

$$\frac{U}{U_0} = -\frac{1}{4C'} \frac{2 + 3\sigma}{1 + \sigma} \tag{4.9}$$

Let $\mathbf{U}_p$ and $\mathbf{U}_f$ denote, respectively, the rate of suspension sedimentation and the fluid velocity in a reference system where the mean volume velocity is $c\mathbf{U}_p + (1 - c)\mathbf{U}_f = 0$. The previously introduced velocity flowing $\mathbf{U}$ onto the sample particle in the system of point particles is related to the relative fluid velocity in a system of particles of finite dimensions is related by the formula

$$\mathbf{U}_f - \mathbf{U}_p = \frac{1 - c(\langle \overline{\mathbf{u}}' \rangle \cdot \mathbf{u}_0)}{1 - c} \mathbf{U}$$

where $\langle \overline{\mathbf{u}}' \rangle$ is the velocity field averaged over the volume of the sample particle determined by formula (1.3) and formally continued into the unit radius sphere.

It can be shown that $(\langle \overline{\mathbf{u}}' \rangle \cdot \mathbf{u}_0) = -3D'$, from which follows that

$$\mathbf{U}_p = -(1 + 3cD')\mathbf{U}, \quad \mathbf{U}_f = (1 + 3cD')c\mathbf{U} / (1 - c)$$

Thus the suspension sedimentation rate is

$$\frac{U_p}{U_0} = \frac{1}{4}\,\frac{2+3\sigma}{1+\sigma}\left[\frac{4-23c+12cI+0.4c^2}{3-2v_0-v_0c}(1-cv_0)+c\left(1-\frac{2}{5}c\right)\right] \quad (4.10)$$

Formula (4.10) makes possible the determination of the suspension sedimentation rate U_p depending on the particle volume concentration c. Note that the ratio U_p / U_0 depends on the binary correlation function $g(r)$ only through the integral I introduced in (4.6).

Convergence of that integral was investigated in [1], where it was established that $g(r)$ exponentially rapidly tends to unity when $r \to \infty$.

It was also noted in [1] that the two-term expansion of $g(r)$ in series in small c is of the form

$$g(r) = \begin{cases} 1+c(8-3r+{}^1/_{16}r^3), & 2<r<4 \\ 1, \quad r>4 \end{cases}$$

For small c that formula yields

$$I = {}^{22}/_5 c \quad (4.11)$$

Using formula (4.10) it is possible to analytically determine the suspension sedimentation rate with an accuracy to within terms of order c^2. For suspensions of solid spheres $(v_0 = 0)$ we have

$$U_p / U_0 = 1 - 5c + 13c^2 \quad (4.12)$$

A considerable number of investigations surveyed in [3] is devoted to the determination of the sedimentation rate of spatially homogeneous suspension of spherical particles randomly distributed in a fluid. The following empirical formula was proposed there;

$$U_p / U_0 = (1-c)^n, \quad n \approx 5 \quad (4.13)$$

which is in satisfactory agreement with (4.12) to within terms of order c^2 for $n = 5$

$$U_p / U_0 = 1 - 5c + 10c^2 - \ldots$$

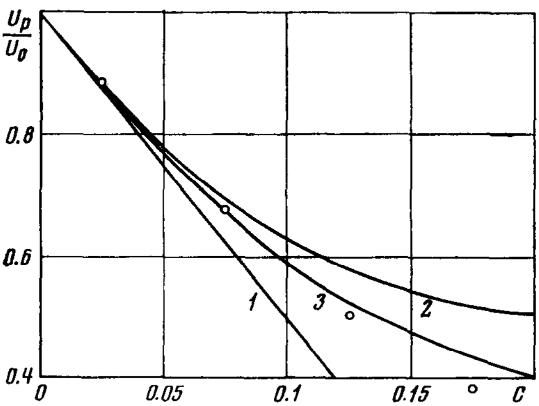

Fig. 1

The use of the binary correlation function in the form of a two-term expansion is justified only for small c. Function $g(r)$ was obtained in [1] by numerical calculations in the range of small and moderate values of c from 0 to 0.2. It made possible to determine I and obtain the ratio U_p / U_0 in that range. The dependence of the calculated ratio U_p / U_0 on volume concentration is shown in Fig. 1. Curve 1 corresponds to the linear formula $U_p / U_0 = 1 - 5c$ and curve 2 to the quadratic formula (4.12).

Curve 3 represents the ratio U_p / U_0 numerically calculated using the binary correlation function determined in [1], and the small circles relate to experimental data determined in [3] which correspond to values of exponent n equal 4.8, 5.0, and 5.6 in formula (4.13). It will be seen that numerically determined values of the ratio U_p / U_0 are in satisfactory agreement with the indicated experimental data.

It should be noted that these results are inapplicable for defining suspension sedimentation in the presence of clustering and in the case of orderly distribution of particles.

In conclusion let us consider the asymptotics of $U_p / U_0 - 1$ when $c \to 0$. Three points of view on the dependence of that expression on c appear to exist in literature. Formula (4.12) derived here implies that for small c $U_p / U_0 - 1 \sim c$.

The same dependence on c was obtained in [4] using the method based on the calculation of the difference between two divergent integrals, but the coefficient 6.55 at c derived there differs somewhat from the one obtained here. It can be shown, however, that, if in [4] and the present paper all particles, except the sample, were assumed to be point particles, both results would be the same: $U_p / U_0 = 1 - 5.5c$. The difference is due to the omission of taking into account that particles are not points and, also, to the rather incorrect allowance in [4] for the interaction between two isolated spheres, as indicated in the introduction.

With ordered distribution of spheres and in models of cells the dependence of sedimentation rate on concentration is, as shown in [2], different; $U_p / U_0 - 1 \sim c^{1/3}$. Dependence of that kind, as indicated in [4], is natural in problems in which one or another kind of boundary conditions are specified at distances of order $ac^{-1/3}$ from the center of the sample particle. With the random distribution of spheres, considered here and in [4], there is no similar linear scale, and this results in a different dependence of $U_p / U_0 - 1$ on c. This subject is considered in greater detail in [4].

An attempt at a theoretical substantiation of Brinkman's formula [11] for the ratio U_p / U_0 was made in [10]. It was shown in [13] that at the limit $c \to 0$

$$U_p / U_0 - 1 \sim \sqrt{c} \tag{4.14}$$

A similar dependence was obtained in [12, 13], where the terms next following with respect to concentration in the expansion of U_p / U_0 were also determined. It was shown in [7] that the problem considered in [10 − 12] corresponds to filtration of fluid through a stationary layer of spherical particles with random, but fixed positions of their centers. In that case the velocity of all particles is the same, and the force acting on each particle is the random quantity. In the problem of free sedimentation the force acting on each particle is known and the same for all particles, hence formula (4.14) is unsuitable for defining the sedimentation of suspension under the action of the force of gravity.

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